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Consensus for discrete-time multi-agent systems with measurement noises based on event-triggered control algorithm[1]

Zhaojun Tang[2]

Abstract. This paper is concerned with event-triggered consensus for discrete-time multi-agent systems with measurement noises. Event-triggered control strategies are employed so as to reduce the frequency of individual control updating. The agents update their controllers only at triggering instants which are determined by a triggering condition. It is also assumed that each agent can only receive noisy measurements of the states of its neighbors. A centralized control strategy is proposed first, and then the results are extended to the decentralized counterpart, in which only the states of its neighbors is required for each agent. The convergence analysis is given with the help of stochastic Lyapunov function and algebraic graph theory. A simulation example is presented to illustrate the theoretical results.

Key words. Consensus, multi-agent system, event-triggered control, measurement noises.

1. Introduction

In recent years, the research on cooperative and coordinated control for multi-agent systems has received considerable attention due to its wide range of potential applications including attitude of spacecraft alignment, formation control of unmanned vehicles, sensor network, and so on. As a critical issue for coordinated control, consensus means that the group of agents reach an agreement on certain quantities of interest. In the past few years, consensus problems have been extensively studied and many profound results have been established.

In practice, the bandwidth of the communication network and the computation resource of the agents in the system are inevitably constrained. Therefore, it is neces-

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[2]School of Science, Chongqing University of Technology, Chongqing, 400054, China

sary to reduce transmission frequency and control actuation update as few as possible in design of control strategy. To deal with such concerns, a commonly used approach is sampled-data control, which means that the information exchange and control input update are executed in a periodic fashion. Consensus with sampled-data control have been reported in many literatures[1, 2]. An important drawback of sampled-data control is that it will lead to conservativeness in the usage of computational resource and bandwidth since the constant sampling period is chosen to guarantee stability for worst case situations. Another interesting sampling method is the event-triggered control, that is, each agent updates its controller at some instances which determined by properly defined events. Event-triggered control strategy seems to be more suitable for cooperative control of multi-agent systems, and it has recently been applied to the consensus problem. In [3], centralized and decentralized event-triggered control strategies are developed for the consensus of first-order multi-agent systems with undirected topology, where the control updating depends on the ratio of a certain measurement error with respect to the norm of a function of the state, and self-triggered control design is also presented to avoid continuous monitoring of the measurement error. The event-triggered consensus of multi-agent systems with weighted and directed topology are investigated in [4, 5]. The event-triggered consensus for multi-agent models including communication delays, nonlinear dynamics, Markovian switching topologies and discrete-time dynamics has also been considered[6, 7, 8].

Note that all the aforementioned literatures assume perfect state exchange among the agents. Obviously, this assumption is often impractical since the information exchange within real networks typically involves quantization, wireless channels and/or sensing. Consensus problems with noisy measurements have attracted the attention of some researchers. Huang and Manton[9] investigated the consensus problem for discrete-time multi-agent systems with fixed topology and noisy measurements, where decreasing consensus gains are introduced in the consensus algorithm in order to attenuate the measurement noises. The results are extended to switching topology in [10]. Mean square average consensus for first-order continuous-time multi-agent systems is studied in [11]. Consensus problems for leader-following multi-agent systems are investigated in [12]. However, the issue of event-triggered consensus for multi-agent systems with noisy measurements receives less attention.

Motivated by the above observations, this paper focuses on the event-triggered consensus for multi-agent systems with measurement noises. That is, it is assumed that each agent can only receive the noisy measurements of the states of its neighbors, and each agent only updates its controller at some triggering instances. A centralized control strategy is firstly proposed to solve the mean square consensus. Based on stochastic Lypunov function and matrix theory technique, a sufficient condition is established to ensure the mean square consensus. Subsequently, the decentralized control strategy is also provided. In addition, decreasing consensus gains are introduced in design of the consensus protocols to attenuate the measurement noises. It is also showed that the Zeno-behavior is excluded since the inter-event time is at least lower bounded by 1.

The following notations will be used throughout this paper. Let I be an identity matrix with appropriate dimension; $\mathbf{1}$ denotes a column vector with all ones. For a given matrix A, A^T denotes its transpose; $\|A\|$ denotes its spectral norm. $\|\cdot\|$ denotes the Euclidean norm for a given vector. For the random variable ξ,$E(\xi)$ denotes its mathematical expectation.

2. Preliminaries

2.1. Algebraic graph theory

For a multi-agent system of N agents, the interaction topology among the agents can be modeled by a diagraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes with node i representing the ith agent, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge e_{ij} in graph $\mathcal{G}$ is denoted by the ordered pair of nodes (i, j), which means that agent j can receive directly the information of agent i. If there is an edge $(i, j) \in \mathcal{E}$, then i is called a neighbor of node j. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. A path in $\mathcal{G}$ from node i_0 to node i_m is a sequence of ordered nodes $i_0, i_1, \ldots, i_m$ with $(i_{j-1}, i_j) \in \mathcal{E}$ for $j = 1, \ldots, m$. An undirected graph is called connected if there is a path between any two nodes of the graph.

The weighted adjacency matrix of the digraph $\mathcal{G}$ is denoted by $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The degree matrix $D = diag\{d_1, d_2, \ldots, d_n\}$ is an $n \times n$ diagonal matrix with $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix associated with the digraph $\mathcal{G}$ is defined as $L = D - A$. It is clear that 0 is an eigenvalue of L, and $\mathbf{1}$ is the corresponding eigenvector. For a connected graph $\mathcal{G}$, the eigenvalues of L can be listed as $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$. Furthermore, for any vector $x \in \mathbb{R}^N$ satisfying $x^T\mathbf{1} = 0$, one has $\lambda_2 x^T x \le x^T L x \le \lambda_N x^T x$[13].

2.2. Problem statement

Consider a multi-agent system including N agents described by

$$x_i(k + 1) = x_i(k) + a(k)u_i(k), i \in \mathcal{V}, k \ge 0, \tag{1}$$

where $a(k) > 0$ is the step size, $x_i(k) \in \mathbb{R}$ and $u_i(k) \in \mathbb{R}$ denote the state and control input of agent i, respectively.

In the present paper, we will investigate the event-triggered consensus control protocol. That is, each agent updates its controller only at triggering instants which are determined by some prescribed triggering condition, and holds constant between two adjacency event-triggered instants. In addition, it is assumed that each agent can only receive noisy measurements of the states of its neighbors. Denote the resulting measurement by agent i of the jth agent's state by

$$y_{ij}(k) = x_j(k) + w_{ij}(k), j \in \mathcal{N}_i, \tag{2}$$

where $w_{ij}(k)$ is the additive noise. It is also assumed that each agent knows its own state exactly.

Definition 1. The agents are said to reach mean square consensus if there is a random variable x^* such that $\lim_{k\to\infty} E[x_i(t) - x^*]^2 = 0, i \in \mathcal{V}$ and $E[x^*]^2 < \infty$.

3. Mean square consensus analysis

In this section, both the centralized and decentralized control strategies for the multi-agent systems are discussed. With these event-triggered strategies, controllers only update at certain triggering instances which are determined by the proposed triggering conditions, and keep steady during the intervals between two triggering instants.

3.1. Centralized strategy

In this subsection, it is assumed that the triggering instants for all agents are the same, i.e., all agents in the system synchronously update their control inputs. In order to achieve consensus, we propose the following consensus protocol

$$u_i(k) = \sum_{j=1}^{N} a_{ij}[y_{ij}(k_l) - x_i(k_l)], \tag{3}$$

where k_l is the lth triggering instant and $k_l \in \{0, 1, \ldots\}$.

Define the measurement error $e_i(k) = x_i(k_l) - x_i(k), k \in [k_l, k_{l+1})$ and the stack vector $e(k) = [e_1(k), e_2(k), \ldots, e_N(k)]^T$. The measurement error $e_i(k)$ shows the difference between the last updated state and the current state. Substituting the protocol (3) into the system (1) leads to

$$x(k+1) = (I - a(k)L)x(k) - a(k)Le(k) + a(k)DW(k_l), \tag{4}$$

where and whereafter $x(k) = (x_1(k), x_2(k), \ldots, x_N(k))^T$; $D = diag(\alpha_1^T, \alpha_2^T, \ldots, \alpha_N^T)$ is an $N \times N^2$ dimensional block matrix with $\alpha_i = (a_{i1}, a_{i2}, \ldots, a_{iN})^T$; $W(k_l) = (w_1^T(k_l), w_2^T(k_l), \ldots, w_N^T(k_l))^T$ with $w_i(k_l) = (w_{i1}(k_l), w_{i2}(k_l), \ldots, w_{iN}(k_l))^T$.

Denote the disagreement vector $\delta(k) = x(k) - \frac{1}{N}\mathbf{1}^T x(k)\mathbf{1}$. We shall show the following event-triggered condition can guarantee the mean square consensus for the multi-agent system (4):

$$\|Le(k)\| < \frac{b}{\|L\|}\|Lx(k)\|, \tag{5}$$

where $0 < b \leq \frac{1}{2}\lambda_2$ is a constant.

Remark 1. The event-triggered condition (5) implies that an event is triggered for the system if the event-triggered condition is violated. More precisely, if $\|Le(k)\| \geq \frac{b}{\|L\|}\|Lx(k)\|$, then the latest triggering instants will be $k_l = k$, and the agents update their controllers. In addition, the measurement errors $e(k)$ are reset to zero vector due to $k = k_l$. Consequently, the measurement errors satisfy the condition $\|Le(k)\| \leq \frac{b}{\|L\|}\|Lx(k)\|$ for any $k \geq 0$.

In order to get the main result, we need the following assumptions:

(A1) The sequence $\{a(k), k \geq 0\}$ satisfies: $\sum_{k=0}^{\infty} a(k) = \infty$ and $\sum_{k=0}^{\infty} a^2(k) < \infty$.

(A2) The noises $\{w_{ij}(k), k \geq 0, i \in \mathcal{V}, j \in \mathcal{N}_i\}$ are independent with respect to the indices i, j, k and $Ew_{ij}(k) = 0$, $\sup_{i,j,k} E|w_{ij}(k)|^2 < \infty$.

(A3) The interaction topology $\mathcal{G}$ is undirected and connected.

Before moving on, we need the following lemma.

Lemma 1.[14] Let $\{u(k), k = 0, 1, \ldots\}, \{\alpha(k), k = 0, 1, \ldots\}$ and $\{q(k), k = 0, 1, \ldots\}$ be real sequence, satisfying $0 < q(k) \leq 1, \alpha(k) \geq 0, k = 0, 1, \ldots, \sum_{k=0}^{\infty} q(k) = \infty$, $\alpha(k)/q(k) \to 0, k \to \infty$, and

$$u(k+1) \leq (1 - q(k))u(k) + \alpha(k).$$

Then $\limsup_{k\to\infty} u(k) \leq 0$. In particular, if $u(k) \geq 0, k = 0, 1, \ldots,$ then $u(k) \to 0$ as $k \to \infty$.

We have the following theorems.

Theorem 1. Apply the protocol (3) to the system (1) with the event-triggered condition (5) and suppose that (A1)-(A3) hold. Then,

$$\lim_{k\to\infty} E[V(k)] = 0, \tag{6}$$

where $V(k) = \frac{1}{2}\delta^T(k)\delta(k)$.

Proof. For convenience, denote $J = \frac{1}{N}\mathbf{1}\mathbf{1}^T$. From (A3), we have $\mathbf{1}^T L = 0$. Then, it follows from (4) that

$$\delta(k+1) = (I - a(k)L)\delta(k) - a(k)Le(k) + a(k)(I - J)DW(k_l). \tag{7}$$

Thus, we have

$$\begin{aligned} V(k+1) = {} & \tfrac{1}{2}\delta^T(k)\delta(k) - a(k)\delta^T(k)L\delta(k) + \tfrac{1}{2}a^2(k)\delta^T(k)L^2\delta(k) - a(k)\delta^T(k)\times \\ & (I - a(k)L)Le(k) + a(k)\delta^T(k)(I - a(k)L)(I - J)DW(k_l) + \tfrac{1}{2}a^2(k)\times \\ & e^T(k)L^2e(k) - a^2(k)e^T(k)LDW(k_l) + \tfrac{1}{2}a^2(k)D^TW^T(k_l)(I - J)DW(k_l) \\ \leq {} & (1 - 2\lambda_2 a(k) + \lambda_N^2 a^2(k))V(k) + a(k)\left\|\delta(k)\right\|\left\|I - a(k)L\right\|\left\|Le(k)\right\| \\ & + \tfrac{1}{2}a^2(k)\left\|Le(k)\right\|^2 + a(k)\delta^T(k)(I - a(k)L)(I - J)DW(k_l) \\ & - a^2(k)e^T(k)LDW(k_l) + \tfrac{1}{2}a^2(k)D^TW^T(k_l)(I - J)DW(k_l), \end{aligned}$$

where we have used the fact that $L(I - J) = L$ and $(I - J)^2 = I - J$. From (A1), we know that $a(k) \to 0$ as $k \to \infty$, which implies that there exists $k_0 > 0$ such that

$$\|I - a(k)L\| \leq 1 \quad \text{and} \quad a(k) \leq \frac{1}{2\lambda_N^2 + \frac{1}{2}\lambda_2^2}. \tag{8}$$

Noting that $\|L\delta(k)\| = \|Lx(k)\|$, it follows from the trigger condition (5) that

$\|Le(k)\| \le \frac{b}{\|L\|}\|L\delta(k)\|$, which leads to $\|Le(k)\| \le b\|\delta(k)\|$. Thus we have

$$\begin{aligned} V(k+1) \le\ & [1-2\lambda_2 a(k)+\lambda_N^2 a^2(k)+2ba(k)+b^2a^2(k)]V(k) \\ & +a(k)\delta^T(k)(I-a(k)L)(I-J)DW(k_l)-a^2(k)e^T(k)LDW(k_l) \\ & +\tfrac{1}{2}a^2(k)W^T(k_l)D^T(I-J)DW(k_l). \end{aligned}$$

Taking mathematical expectation on both sides of the above equality, from (A2) and (9) we can get

$$E[V(k+1)] \le [1-\frac{1}{2}\lambda_2 a(k)]E[V(k)]+\frac{1}{2}a^2(k)\|D\|^2\|I-J\|N^2\sigma_W,$$

where $\sigma_W = \sup_{i,j,k} E|w_{ij}(k)|^2$.

By (A1) and Lemma 3.1 we have $\lim_{k\to\infty} E[V(k)] = 0$. Therefore, the proof is completed.

Theorem 2. Apply the protocol (3) to the system (1) with the event-triggered condition (5) and suppose that (A1)-(A3) hold. Then the n agents reach mean square consensus.

Proof. It follows from (4) and $\mathbf{1}^T L = 0$ that

$$\frac{1}{N}\sum_{i=1}^{N} x_i(k+1) = \frac{1}{N}\sum_{i=1}^{N} x_i(k) + \frac{1}{N}a(k)\mathbf{1}^T DW(k_l), \tag{9}$$

which leads to

$$\frac{1}{N}\sum_{i=1}^{N} x_i(n) = \frac{1}{N}\sum_{i=1}^{N} x_i(0) + \frac{1}{N}\mathbf{1}^T D\sum_{k=0}^{n-1} a(k)W(k_l).$$

By the martingales convergence theorem [15], it follows that $\sum_{i=1}^{n} a(k)W(k_l)$ converges in mean square as $n\to\infty$. Then $\sum_{i=1}^{\infty} a(k)W(k_l)$ is well defined. Denote

$$x^* = \frac{1}{N}\sum_{i=1}^{N} x_i(0) + \frac{1}{N}\mathbf{1}^T D\sum_{k=0}^{\infty} a(k)W(k_l).$$

We have

$$E(x^*) = \frac{1}{N}\sum_{i=1}^{N} x_i(0).$$

and

$$Var(x^*) = \frac{1}{N^2}\sum_{k=0}^{\infty}\left[a^2(k)E\left(\sum_{i,j} a_{ij}w_{ij}(k_l)\right)^2\right].$$

Thus, we have

$$\begin{aligned} E(x^*)^2 &= \left(\tfrac{1}{N}\sum_{i=1}^{N} x_i(0)\right)^2 + Var(x^*) \\ &\leq \left(\tfrac{1}{N}\sum_{i=1}^{N} x_i(0)\right)^2 + \sigma_W \sum_{i,j} a_{ij}^2 \sum_{k=0}^{\infty} a^2(k). \end{aligned}$$

In light of (A1), we have $E(x^*)^2 < \infty$, which together Theorem 1 lead to the proof of the theorem.

Remark 2. It is easily seen that the inter-event time is at least lower bounded by 1, which implies that Zeno-behavior is absolutely excluded.

3.2. Distributed strategy

It should be noted that the event-triggered condition (5) is centralized since the global information is required. Hence, the consensus protocol (3) is not fully distributed. In this subsection, we will propose a distributed triggering condition. In this scenario, the triggering instants for different agents may be different. We propose the following consensus protocol

$$u_i(k) = \sum_{j=1}^{N} a_{ij}[y_{ij}(k_{l'}^j) - x_i(k_l^i)], i \in \mathcal{V}, \tag{10}$$

where $k_{l'}^j = \max\{k^j | k^j \in \{k_l^j, l = 0, 1, \ldots\}, k^j \leq k\}$, $k_l^i = \max\{k^i | k^i \in \{k_l^i, l = 0, 1, \ldots\}, k^i \leq k\}$ which represent the latest event-triggered instants for the agents j and i, respectively.

Substituting the protocol (10) into the system (1), we have

$$x_i(k+1) = x_i(k) + a(k) \sum_{j=1}^{N} a_{ij}[y_{ij}(k_{l'}^j) - x_i(k_l^i)]. \tag{11}$$

The measurement error for agent i is defined as

$$e_i(k) = x_i(k_l^i) - x_i(k), k \in [k_l^i, k_{l+1}^i),$$

which leads to $x_j(k_{l'}^j) = x_j(k) + e_j(k)$. Then (11) can be rewritten as

$$x_i(k+1) = x_i(k) + a(k) \sum_{j=1}^{N} a_{ij}[x_j(k) - x_i(k) + e_j(k) - e_i(k) + w_{ij}(k_{l'}^j)], \tag{12}$$

which can also be written in vector form as

$$x(k+1) = (I - a(k)L)x(k) - a(k)Le(k) + a(k)DW(k_{l'}), \tag{13}$$

where $W(k_{l'}) = (w_1^T(k_{l'}), \ldots, w_N^T(k_{l'}))^T$ with $w_i(k_{l'}) = (w_{i1}(k_{l'}^1), \ldots, w_{iN}(k_{l'}^N))^T$. In order to achieve consensus, we propose the following decentralized event-triggered condition

$$|e_i(k)| < \frac{b}{\|L\|^2}|z_i(k)|, \tag{14}$$

where $z_i(k) = \sum_{j=1}^{N} a_{ij}[x_j(k) - x_i(k)]$, $0 < b \leq \frac{1}{2}\lambda_2$ is a constant.

Remark 3. It is clear that the condition (14) is decentralized since only local measurement error and the neighbors' states for each agent is required. If the triggering condition (14) is violated, i.e., $|e_i(k)| \geq \frac{b}{\|L\|^2}|z_i(k)|$, then agent i update its controller, and the new triggering instant will be $k_l^i = k$.

Theorem 3. Apply the protocol (10) to the system (1) with the event-triggered condition (14) and suppose that (A1)-(A3) hold. Then,

$$\lim_{k\to\infty} E[V(k)] = 0, \tag{15}$$

where $V(k) = \frac{1}{2}\delta^T(k)\delta(k)$ with $\delta(k) = (I - \frac{1}{N}\mathbf{1}\mathbf{1}^T)x(k)$.

Proof. It follows from (13) that

$$\delta(k+1) = (I - a(k)L)\delta(k) - a(k)Le(k) + a(k)(I - J)DW(k_{l'}). \tag{16}$$

Thus, we have

$$\begin{aligned} V(k+1) &= \tfrac{1}{2}\delta^T(k)\delta(k) - a(k)\delta^T(k)L\delta(k) + \tfrac{1}{2}a^2(k)\delta^T(k)L^2\delta(k) - a(k)\delta^T(k)\times \\ &\quad (I - a(k)L)Le(k) + a(k)\delta^T(k)(I - a(k)L)(I - J)DW(k_{l'}) + \tfrac{1}{2}a^2(k)\times \\ &\quad e^T(k)L^2e(k) - a^2(k)e^T(k)LDW(k_{l'}) + \tfrac{1}{2}a^2(k)D^TW^T(k_l)(I - J)DW(k_{l'}) \\ &\leq (1 - 2\lambda_2 a(k) + \lambda_N^2 a^2(k))V(k) + a(k)\|\delta(k)\|\,\|I - a(k)L\|\,\|Le(k)\| \\ &\quad + \tfrac{1}{2}a^2(k)\|Le(k)\|^2 + a(k)\delta^T(k)(I - a(k)L)(I - J)DW(k_{l'}) \\ &\quad - a^2(k)e^T(k)LDW(k_{l'}) + \tfrac{1}{2}a^2(k)D^TW^T(k_l)(I - J)DW(k_{l'}), \end{aligned}$$

Note that the triggering condition (14) implies that $\|Le(k)\| \leq \frac{b}{\|L\|}\|Lx(k)\|$ which is the same as the condition (5). Thus, the rest proof is similar to that of Theorem 1.

Theorem 4. Apply the protocol (10) to the system (1) with the event-triggered condition (14) and suppose that (A1)-(A3) hold. Then the n agents reach mean square consensus.

Proof. The proof is identical to that of Theorem 2 and hence we omit it.

Remark 4. Both the consensus algorithm (10) and the event-triggered condition (14) is decentralized since only the information of its neighbors' is required for each agent. Nevertheless, the proposed control strategy just reduce the number of the control actuation updates, and each agent need to acquire the states of its neighbors' at each time instance.

4. Numerical examples

In this section, a numerical example is presented to illustrated the theoretical results. Consider a multi-agent system consisting of five agents, and the communication topology $\mathcal{G}$ is shown in Fig. 1. For simplicity, assume that all the weights are set as one. In addition, assume that the variance of the i.i.d zero mean Gaussian measurement noises is $\sigma^2 = 0.01$ and the step size $a(k) = 1/(k+1), k > 0$. It is clear that Assumptions (A1)-(A3) hold. By simple calculation, we obtain $\lambda_2 = 0.8299$, and we choose $b = 0.4$.

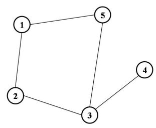

Fig. 1. The communication topology $\mathcal{G}$.

Consider the decentralized control strategy proposed in Theorem 2. Fig. 2 shows the states trajectories of the agents. It can be seen that the mean square consensus is achieved.

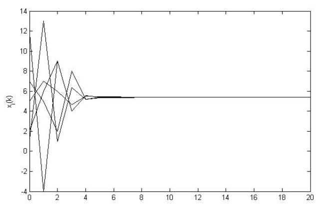

Fig. 2. The state trajectories of the agents with decentralized control strategy.

5. Conclusion

In this paper, we consider event-triggered consensus for multi-agent systems with measurement noises. In order to attenuate the measurement noises, decreasing consensus gains are introduced in the consensus algorithms. A centralized control strategy is proposed first and then the results are extended to the distributed counterpart, in which only the states of its neighbors is required for each agent. It is showed that mean square consensus can be guaranteed with the proposed control strategies provided the network topology is connected. The convergence analysis is given based on stochastic Lypunov function and algebraic theory. Future work will focus on the directed and switching topologies. In addition, it is interesting to design new triggered condition for reducing both the control actuation updates and the communication updates.

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